HAMON TRANSFER STANDARDS

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Abstract

When standard resistors are calibrated against the primary standard, each stage of the measurement must operate at the same power or self-heating changes their resistance values and the calibration cannot be transferred to the next stage. To transfer the accuracy of the UME primary resistance standard to the calibration chain, a Hamon resistor transfer standards is essential.

If each resistor in the Hamon standard resistor are equal in value to 1 in 10^4, then the ratio R_s / R_p is accurate to 10^{-8}. Hamon networks transfer calibration from the reference to the primary standards with short term accuracy much better than the long term accuracy to which equipment is certified. The transfer measurements depend only on short term stability i.e. the few minutes for the measurement.

A very important feature of the Hamon network is that the power dissipated in the individual main resistor is the same in both the parallel and series configurations. This means that there is no need to consider the power coefficient between the two configurations.

1. INTRODUCTION

Practicalities of setting up a national calibration chain

At UME the method of disseminating the calibration of the primary standard to the rest of the National standards is carried out by ratio comparison using direct current comparator bridge (9975). The operation principle of the bridge is described below [1] in Figure 1.

The measurements are performed in 10:1 ratio mode to obtain the best measurement resolution. The current I_x is set by the operator, and the current I_s is generated by the instrument. This produces a problem in the measurement [2] as indicated in the schematic diagram of figure 2.

Calibrating the lower resistance standards with the direct ratio path requires that different current inputs are used and hence different powers dissipated in the 1kΩ primary standard in the R_x and R_s modes of operation. Different powers cause a change in the resistance value, due to self heating. This means that the resistance value of the 1kΩ laboratory standard is unknown in the second ratio measurement and thus calibration of the lower decade resistor values cannot proceed. In order to proceed with the calibration, Hamon transfer standards must be employed in the calibration chain.

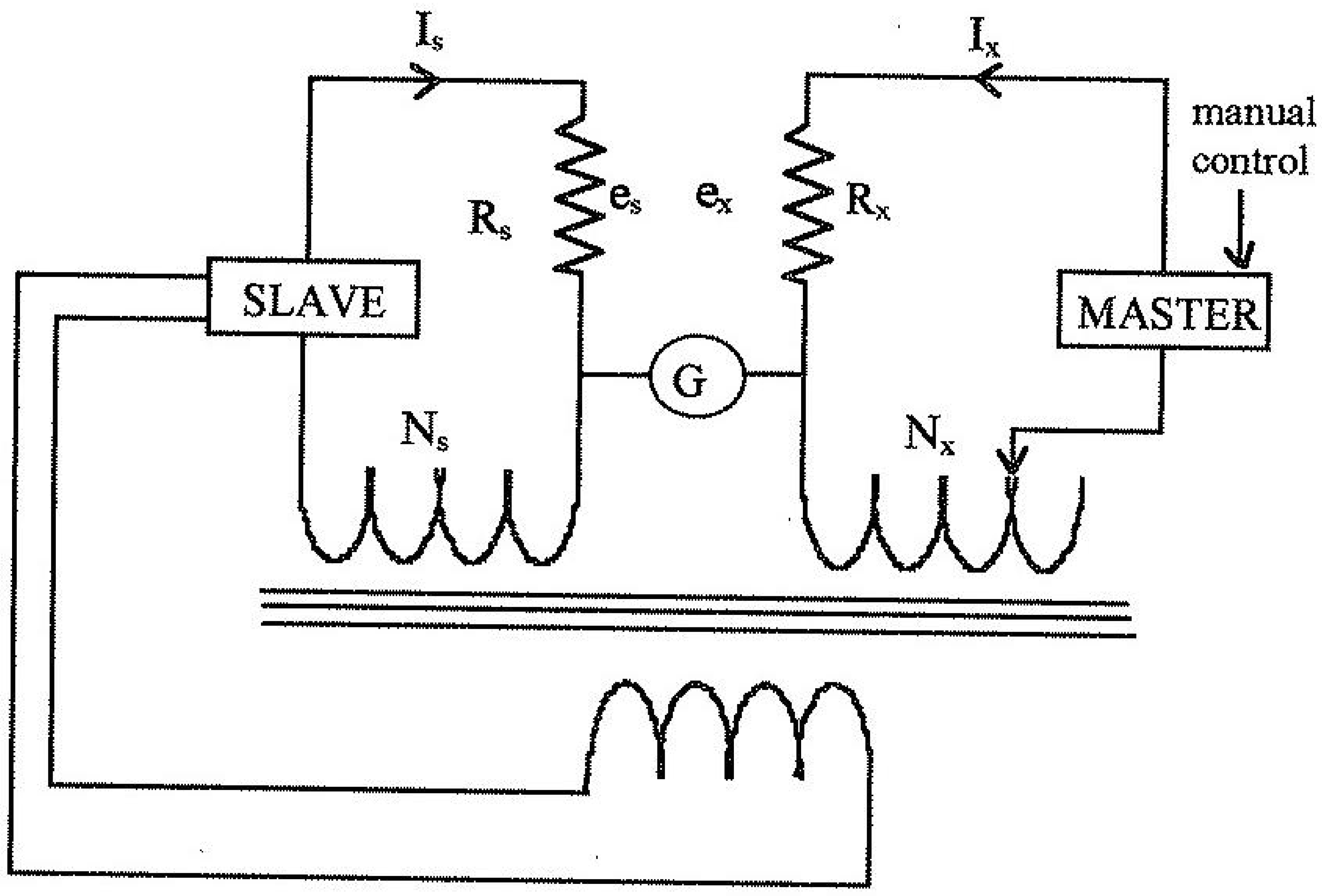


Figure 1 Direct current comparator bridge

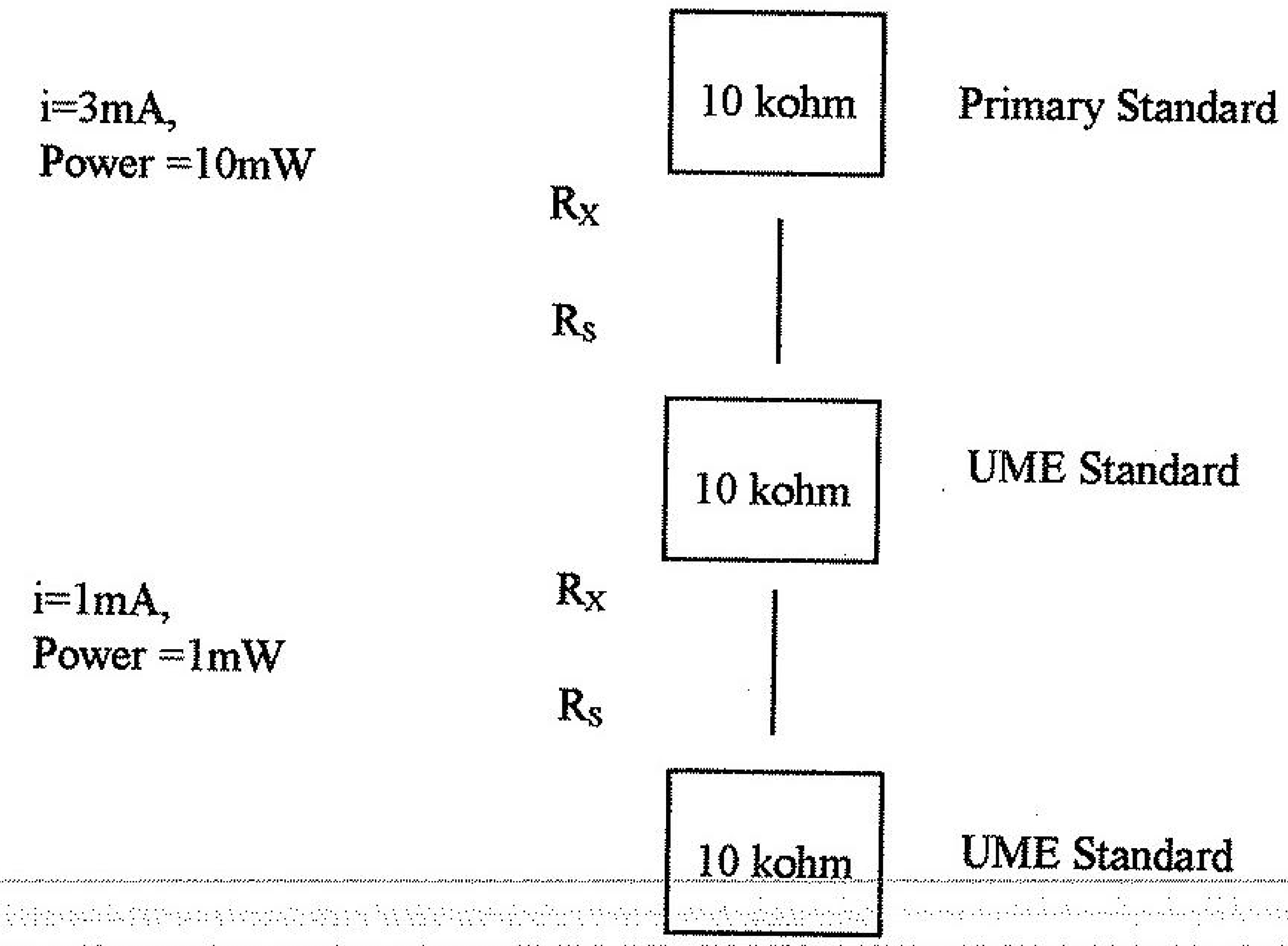


Figure 2 Direct ratio method of comparison

Hamon transfer standards are a resistance network, they comprise a group of ten precision resistors which can be connected both in a series and parallel configuration to achieve very accurate 100:1 and 10:1 scaling of resistor values.

2. THEORY

Construction of a standard resistor by series connections

If ten four terminal resistors are connected in series, the final value is the sum of the individual four terminal resistors, however link resistors are also present as shown below. These will contribute to the final resistance value. The main problem is that the link resistor sets up a potential difference across the link, the solution to this problem is to eliminate this potential difference. This can be achieved by connecting the resistors with copper junctions of symmetric shapes[3]. Usually symmetric squares are used as shown below,

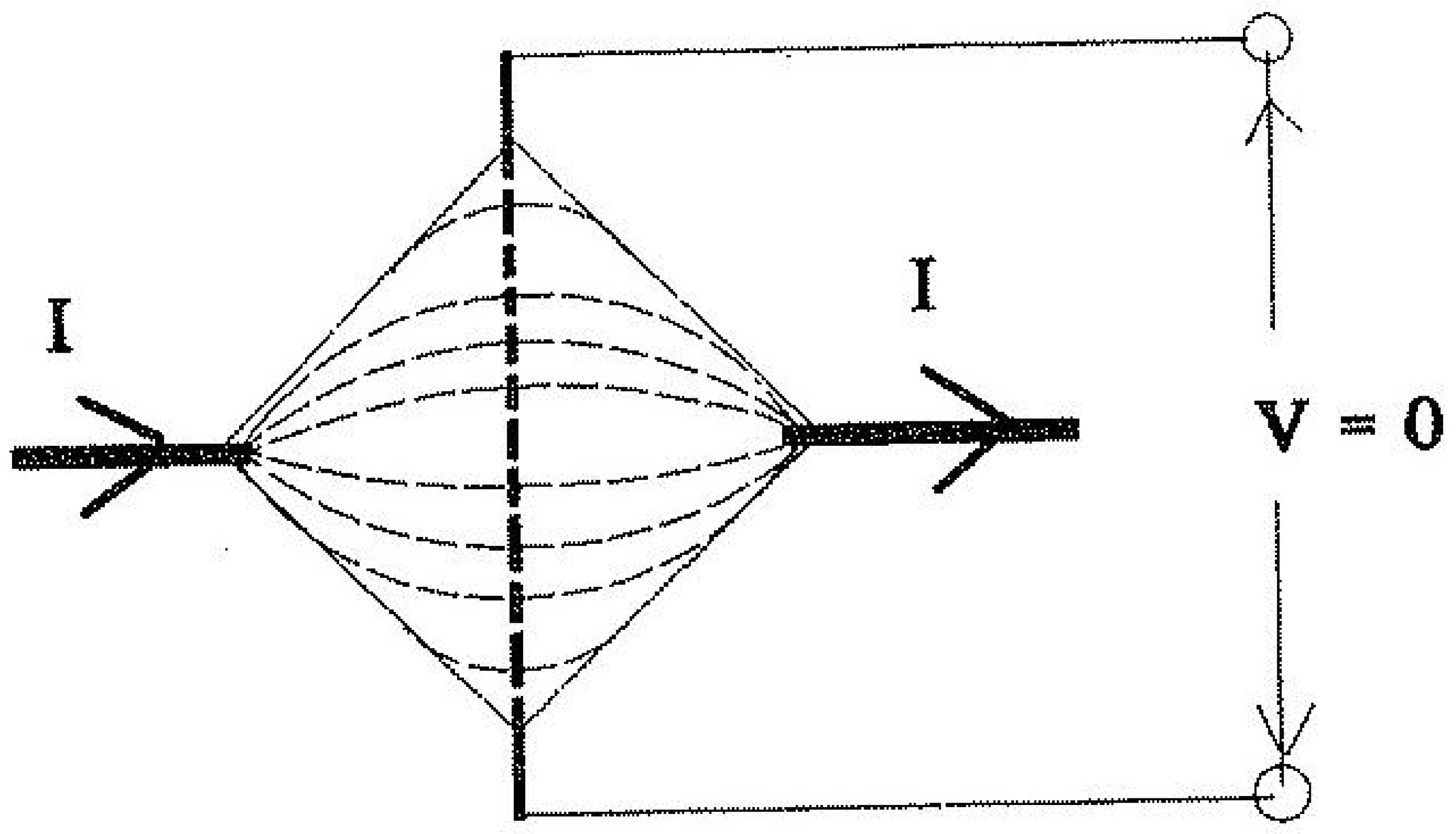


Figure 3 Two dimensional zero resistance junction

The electrons spread out in the pattern above by dashed line. Each electron path length experiences a slightly different resistance by virtue of the different path lengths. The longer the path length the greater the resistance and hence potential. If the square is of sufficient symmetry, the electric fields contributions from the various electron paths throughout the surface and volume cancel out along the line of symmetry indicated by the dashed line. The dashed line is, thus, an equipotential line and potential difference measured along any two points on this line is zero.

Also the following shapes have inherent symmetry properties and can also be used as junctions.

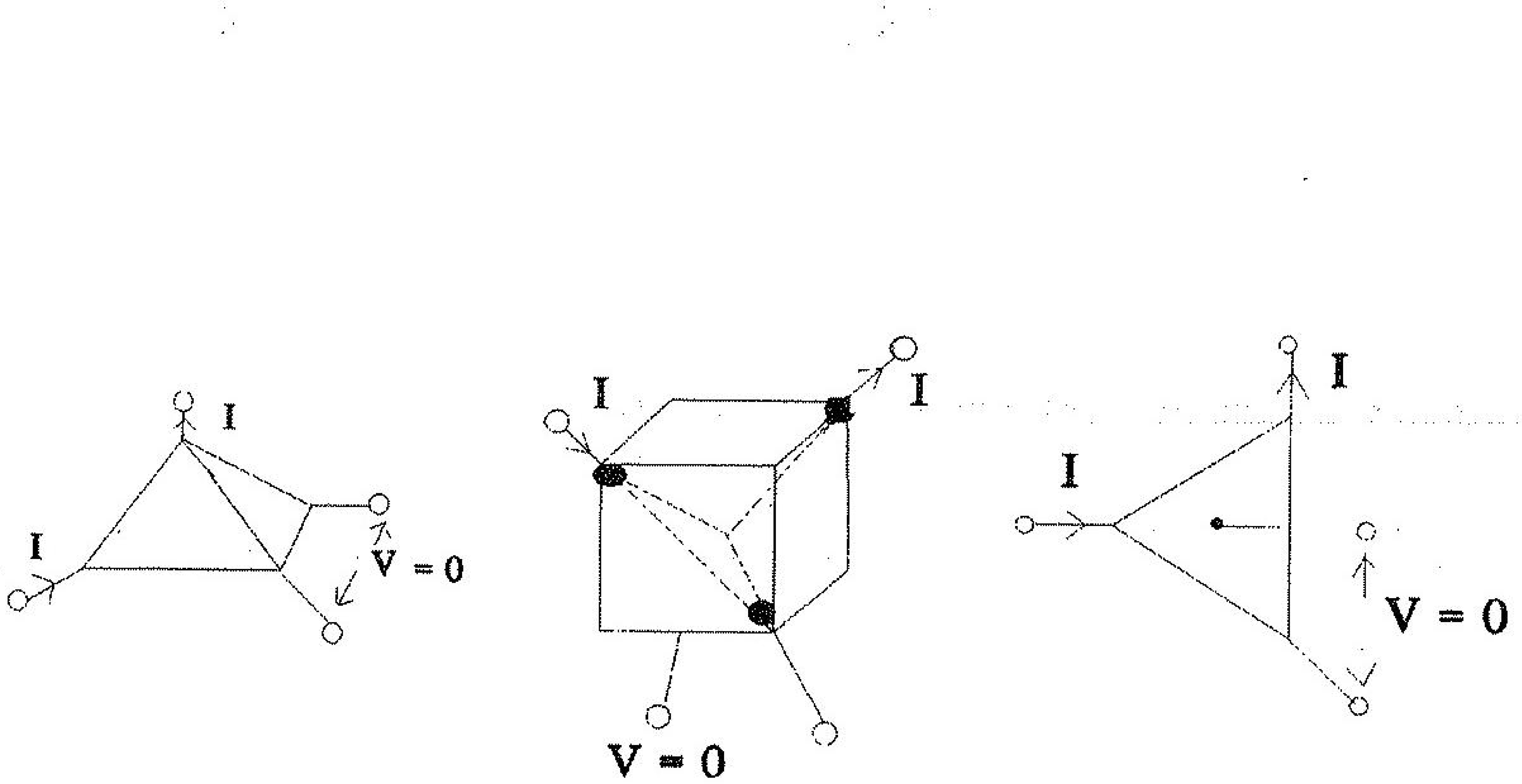


Figure 4 Symmetry properties of a variety of solid conducting junctions.

In all of the above cases the current flowing from the LO current terminal of one resistor, to the HI current terminal of the other resistor spreads out in a symmetrical pattern[4]. All of the above symmetric shapes have equipotentials either in one or two dimensions, as indicated by the dashed lines. If the current is led into and out of the corners indicated, the potential sensed across the remaining two corners will be zero regardless of how large the value of the current is. Thus, R= V/I = 0. This is a zero resistance junction.

The symmetry of the squares is important and is reflected in length, breadth and constant thickness of the junction. If the symmetry properties are not good enough, then the path differences across either of the three dimensional parameters are different and the current paths will not cancel out completely to give constant potential along the middle of the junction.

Typical additive resistance may be 1mΩ, which is a 1ppm error in a 1kΩ resistor. If 10 resistors are connected in series this represents a 10ppm error in the standard. At lower resistance values the symmetry is even more critical. If one is constructing at a much higher resistance range, the same degree of symmetry is not important. The thicker the junction the more volume the current has to flow in. Slight variations in depth at a greater thickness only have small consequence on the symmetry properties, compared with the same variation in a thinner junction. The symmetric junction must be machined to have a thickness which is large enough that the symmetry properties are not upset. A 10% upset in the symmetry properties can lead to a potential resistance of a few micro Ohms. This can be reduced significantly to $10^{-5}\Omega$ with a junction of greater thickness.

Construction of a standard resistor by parallel connections

The problem of connecting four terminal resistances in parallel, so that their combined resistance does not include any link resistors is more complicated. Consider the diagram of Figure 5.

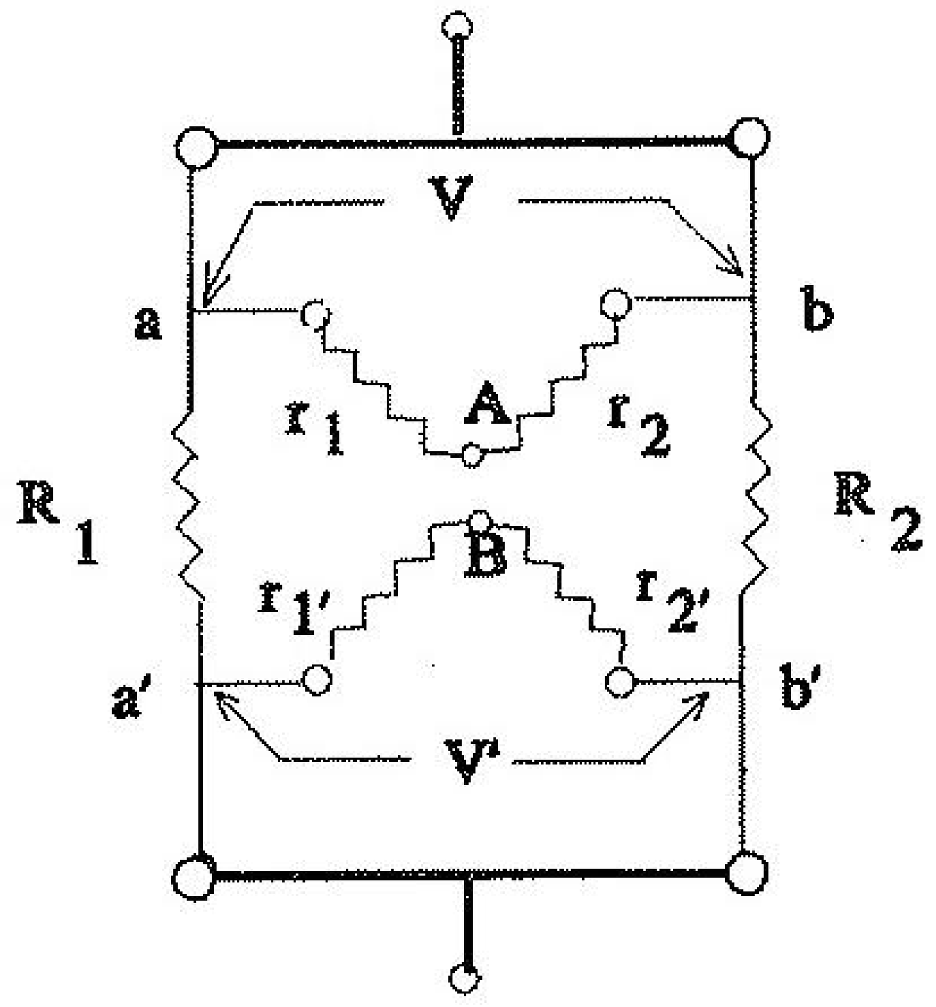


Figure 5 The use of a combining network to achieve a zero resistance Parallel junction

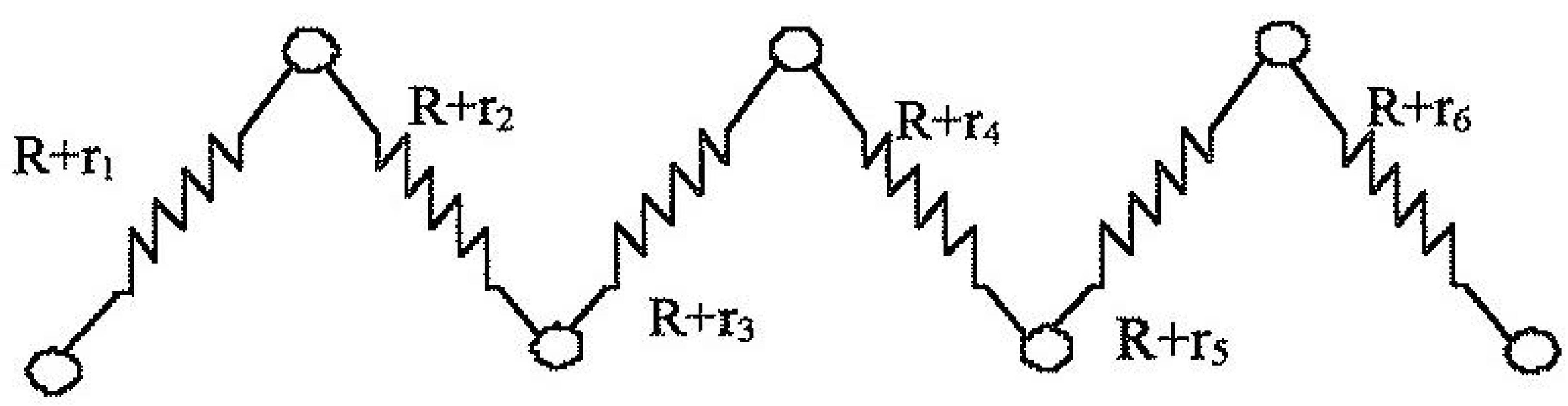
The current terminals have been connected with a low resistance link and the potential terminals by a combining network of two resistors, whose values are large compared to the link resistance, but not so large that they introduce unnecessary resistance into the potential connections of the device. We would like the device to behave as if V and V' were zero. If a detector were connected between A and B, it would be nulled when

$$r_1 / r_2 = R_1 / R_2 \text{ and } r_1' / r_2' = R_1 / R_2 \qquad 1$$

The potential between A and B is independent of the magnitude of V and V' and they can be considered as being zero. The resistors behave as if they were connected in parallel at their internal defining points to yield an accurate value of their individual four terminal resistors[4].

Comparing parallel and series ratios - Hamon standards

Using the last two techniques for building up parallel and series resistors, a resistance network can be constructed of ten nominally equal precision resistors which can be connected in series and in parallel. Such a device is called a Hamon transfer standard.



r_q is the difference from nominal value

Figure 6 Series connection of Hamon standards

In figure 6 ten nominally equal resistors of values, ($R + r_1$, $R + r_2$,... ; R = average value) are connected in series. The total resistance is thus,

$$R_s = 10R + \sum_{q=1}^{10} r_q = 10R \qquad 2$$

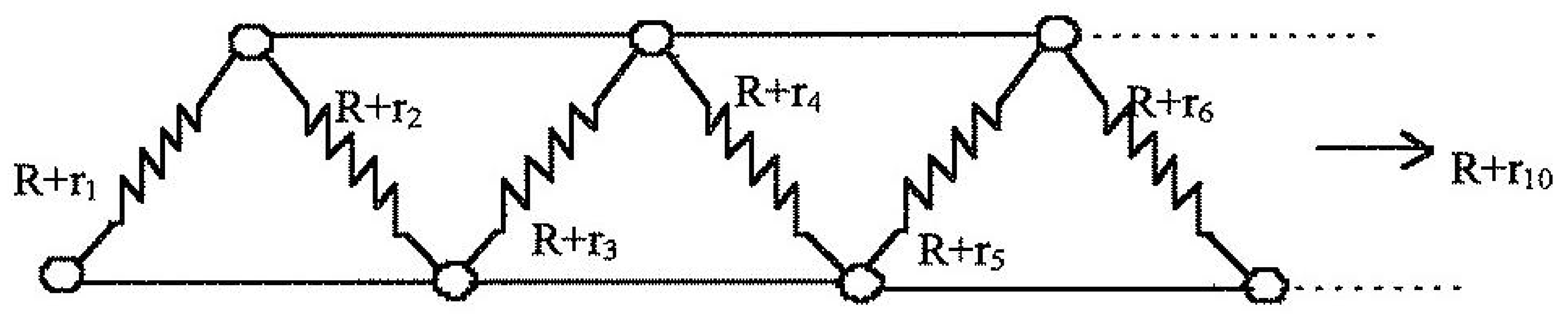


Figure 7 Parallel connection of Hamon standards

In Figure 7 the resistors are connected in parallel, the corresponding auxiliary bridge is shown in Figure 8. Their total resistance is,

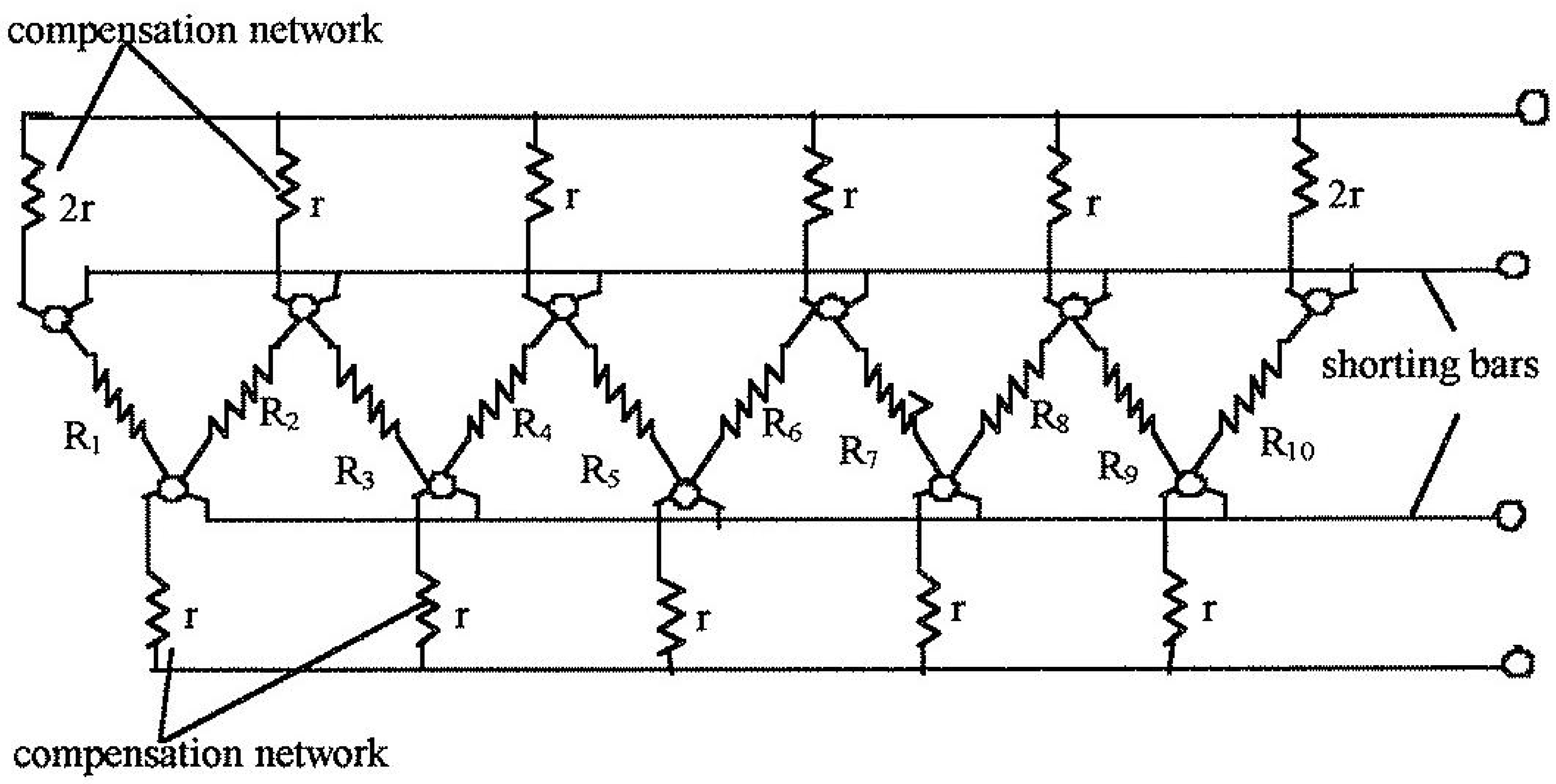


Figure 8 The compensation network of auxiliary bridges

$$\frac{1}{R_p} = \sum_{q=1}^{10} \frac{1}{R + r_q} = \frac{1}{R} \sum_{q=1}^{10} \frac{1}{1 + \frac{r_q}{R}} \qquad 3$$

$$= \frac{1}{R} \sum_{q=1}^{10} \left\{ 1 - \frac{r_q}{R} + \left(\frac{r_q}{R}\right)^2 - \left(\frac{r_q}{R}\right)^3 \ldots\ldots \right\} \approx \frac{10}{R} \qquad 4$$

Since r_q is always less than 100ppm, r_q^2 will be 0.1ppm, thus terms higher than r_q are insignificant, thus,

$$\frac{1}{R_p}=\frac{1}{R}\sum_{q=1}^{10}\left\{1-\frac{r_q}{R}\right\}$$

5

expanding the terms in brackets,

$$\frac{1}{R_p}=\frac{10}{R}-\frac{1}{R^2}\sum_{q=1}^{10}r_q \qquad 6$$

The ratio of series to parallel resistors is found by multiplying equation 2 by equation 6, giving,

$$\frac{R_s}{R_p}=\left(10R+\sum_{q=1}^{10}r_q\right)\left(\frac{10}{R}-\frac{1}{R^2}\sum_{q=1}^{10}r_q\right) \qquad 7$$

multiplying the two terms in brackets and rearranging gives,

$$=(10)^2-\frac{10}{R}\sum_{q=1}^{10}r_q+\frac{10}{R}\sum_{q=1}^{10}r_q-\frac{1}{R^2}\left(\sum_{q=1}^{10}r_q\right)^2 \qquad 8$$

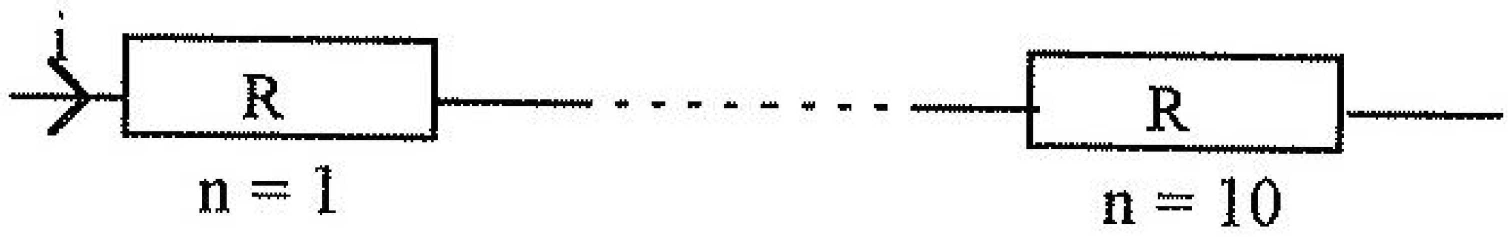
Thus,

$$\frac{R_s}{R_p}=(10)^2-\sum_{q=1}^{10}\left(\frac{r_q}{R}\right)^2 \qquad 9$$

This means that if the individual resistors are equal in value to 1 in 10^4, then the ratio of R_s / R_p is accurate to 1 part in 10^8. For instance ten nominal 100Ω resistors can be connected in parallel to give a nominal 10Ω resistor and then reconnected in series to give a nominal 1kΩ resistor. Thus, the series value is 100 times greater than the parallel value to an accuracy of 1 part in 10^8. Although the accuracy of the series to parallel ratio is very good. The individual resistors are only precision resistors, and are thus subject to drifts in their resistance values with time. Their drifts over large amounts of time (i.e. one week) are very high compared with those of standard resistors. However, in the short time it takes to carry calibration (a few minutes) their absolute drift is below their series-parallel transfer accuracy of 1 in 10^8.

Power dissipation in 100:1 and 10:1 Hamon scaling resistances[5]

In Hamon networks equal powers are dissipated in individual resistors in both configurations. In addition it is also possible to connect the resistors in a series-parallel configuration, which is ten times lower than the series connection. The relationships between their relative currents can be derived. Figures 9 to 11 show their relative configurations.



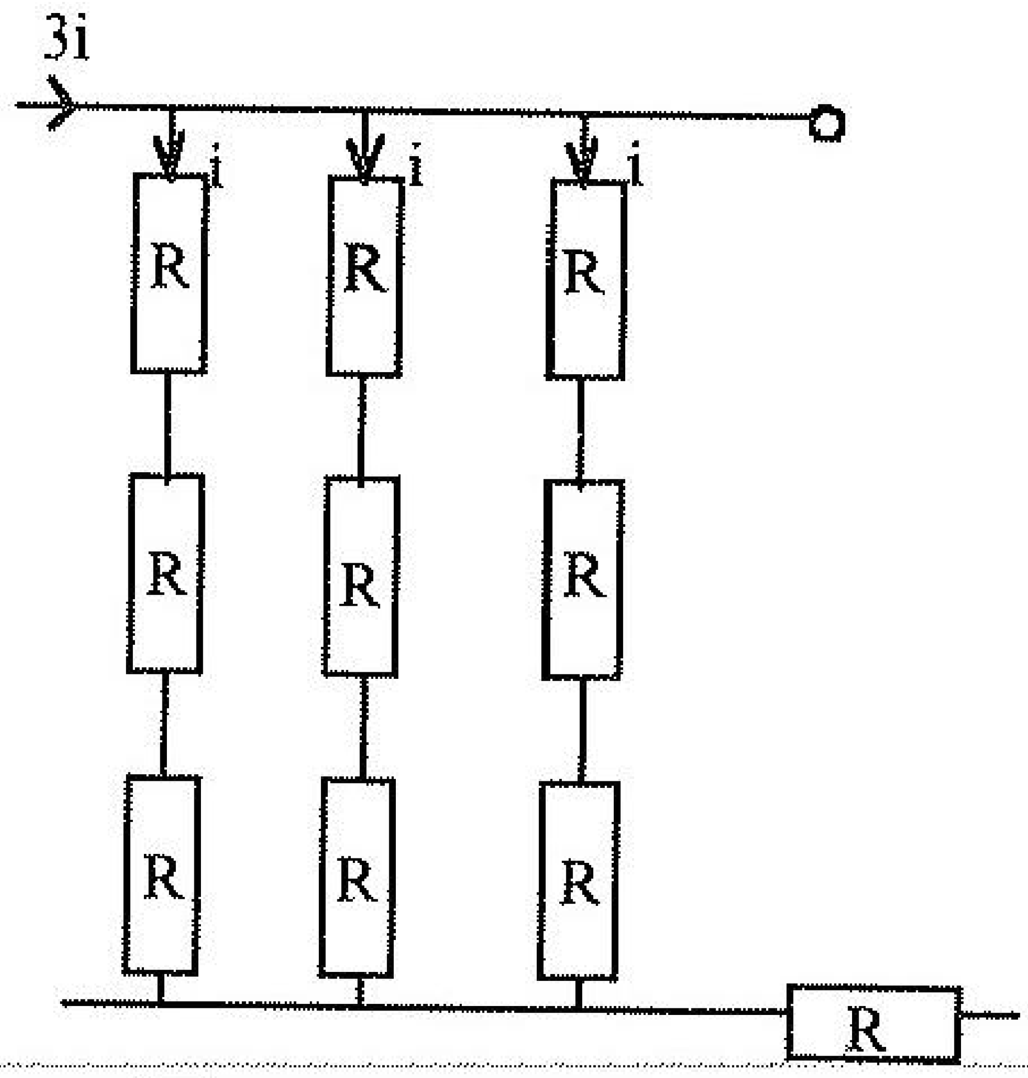
$$i = \sqrt{\frac{P}{R}}$$

i=current

P=power

R=resistor value

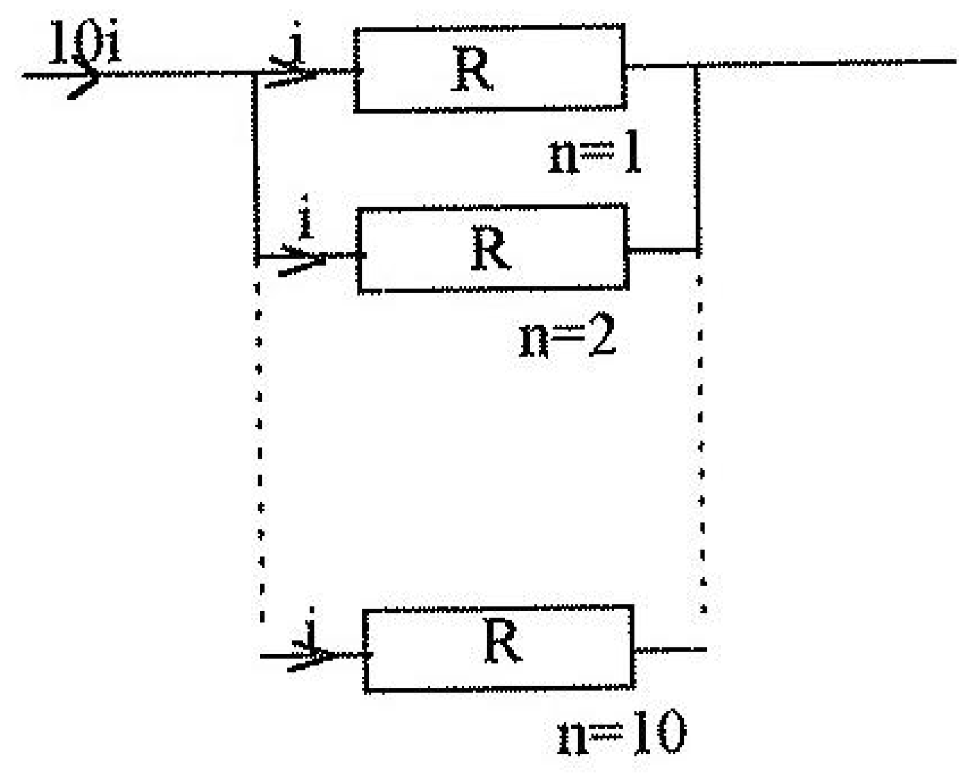
Figure 9 Current flowing in series connections



$$i = 3\sqrt{\frac{P}{R}} + \Delta$$

Δ is a small error terms to account for the tenth unused resistor.

Figure 10 Current flowing in series-parallel connections



$$i = 10\sqrt{\frac{P}{R}}$$

Figure 11 Current flowing in parallel connections

The ratio of currents flowing in the individual resistors, which cause equal powers to be dissipated in the three configurations are as given below.

Series : Series-Parallel : Parallel

1 : 3 : 10

Setting Up The UME Calibration Chain

At UME rather than using a direct ratio method for calibration, a substitution method is used. The principles of this method are,

(i) The highest bridge ratio of 10:1 must always be used to attain the best resolution.

(ii) The calibration proceeds as a two step process using a buffer resistor. This eliminates errors due to systematic errors in the bridge. This process is shown in the schematic of Figure 12

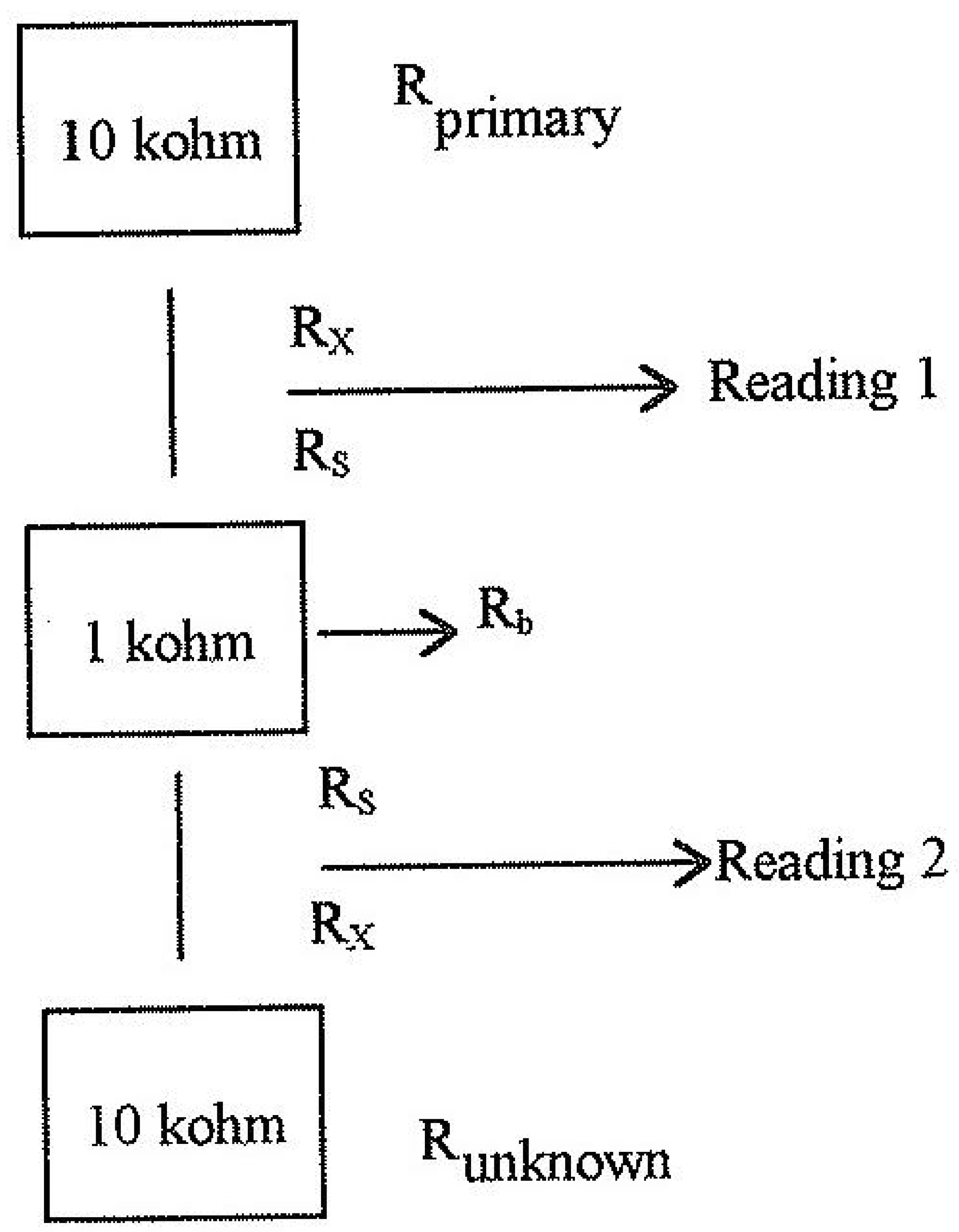


Figure 12 Use of the substitution method for calibration.

The calculation of the unknown resistor ($R_{unknown}$) from the Primary standard resistor ($R_{primary\ standard}$) using a buffer resistor (R_b) is as shown below,

$$\frac{R_{primary\ standard}}{R_b} = \text{Reading}_1 \tag{10}$$

$$\frac{R_{Unknown}}{R_b} = \text{Reading}_2 \tag{11}$$

Multiplying the two ratio gives,

$$\frac{R_{primary\ standard}}{R_{Unknown}} = \frac{\text{Reading}_1}{\text{Reading}_2} \tag{12}$$

From this the unknown resistance is calculated as,

$$R_{Unknown} = R_{Primary\ Standard} \times \frac{R_2}{R_1} \tag{13}$$

In this way, using the Hamon standards a calibration chain can be set-up to accurately transfer the uncertainty of both the 10kΩ and 1Ω two UME primary standards to the high and low value resistance standards.

3. RESULTS

In the calibration chains below the Hamon resistor transfer standards are calibrated using a buffer resistor to keep the highest resolution ratio of 10:1. The Hamon standard is converted to either a parallel configuration or a series-parallel configuration. The value of the Hamon standard is then 1/100 and 1/10 respectively of the calibrated series value, to an accuracy of 1 in 10^8.

Low resistance calibration chain

The low resistance calibration chain is shown in Figure 13. In this range a 100Ω UME resistor standard is used to check the high resistance chain. It is calibrated in the high resistance calibration chain. The two calibrated values can then be compared with each other as a check of the integrity of the two calibration chains.

High resistance calibration chain

The high value resistance calibration chain is shown in Figure 14. In the high resistance calibration chain the bridge comparator operated in a constant voltage mode of operation from 10kΩ upwards. However, the power dissipation requirements are used to match the voltage needed to avoid self heating effects, which chance the value of the resistance.

In Figure 14 for resistor values above 10kΩ, the equivalent currents rather than the voltages used are shown, as an aid to understanding.

4. DISCUSSION AND CONCLUSION

Resistance calibration chains have been set-up for the first time in UME. They will be used to successfully disseminate the unit of Ohm with 1.5ppm uncertainty to the Turkish national standards. These were up until recently only known at uncertainty calculated from the resistance-time graphs of 20ppm. This poor uncertainty disseminated was due to the points mentioned of, not taking the power coefficients into account and not making full use of the bridge resolution. These problems can be remedied by the use of the substitution method of calibration and dissemination of uncertainties using Hamon transfer standards.

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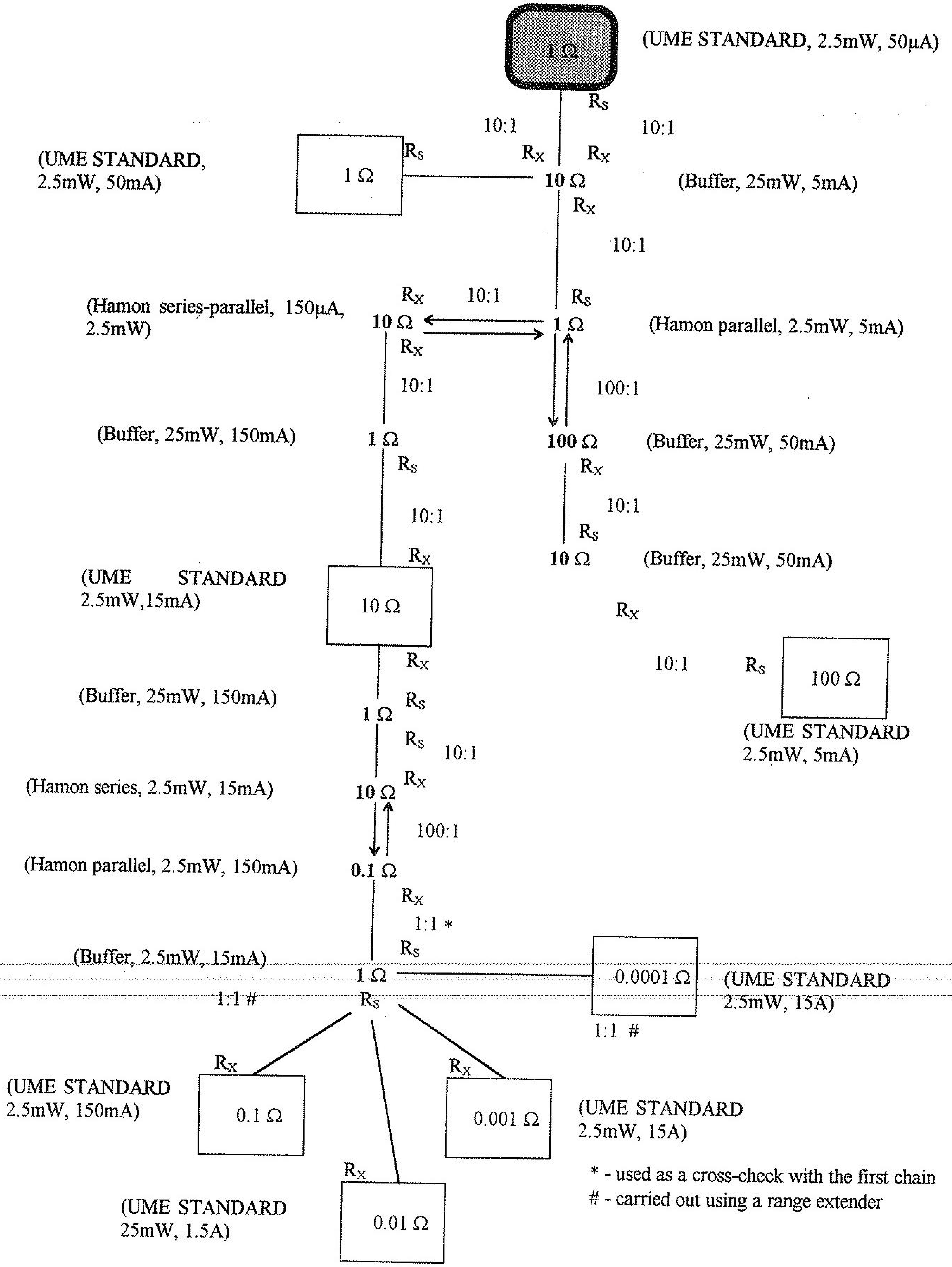

Figure 13 Resistance calibration chain from 100μΩ to 100Ω

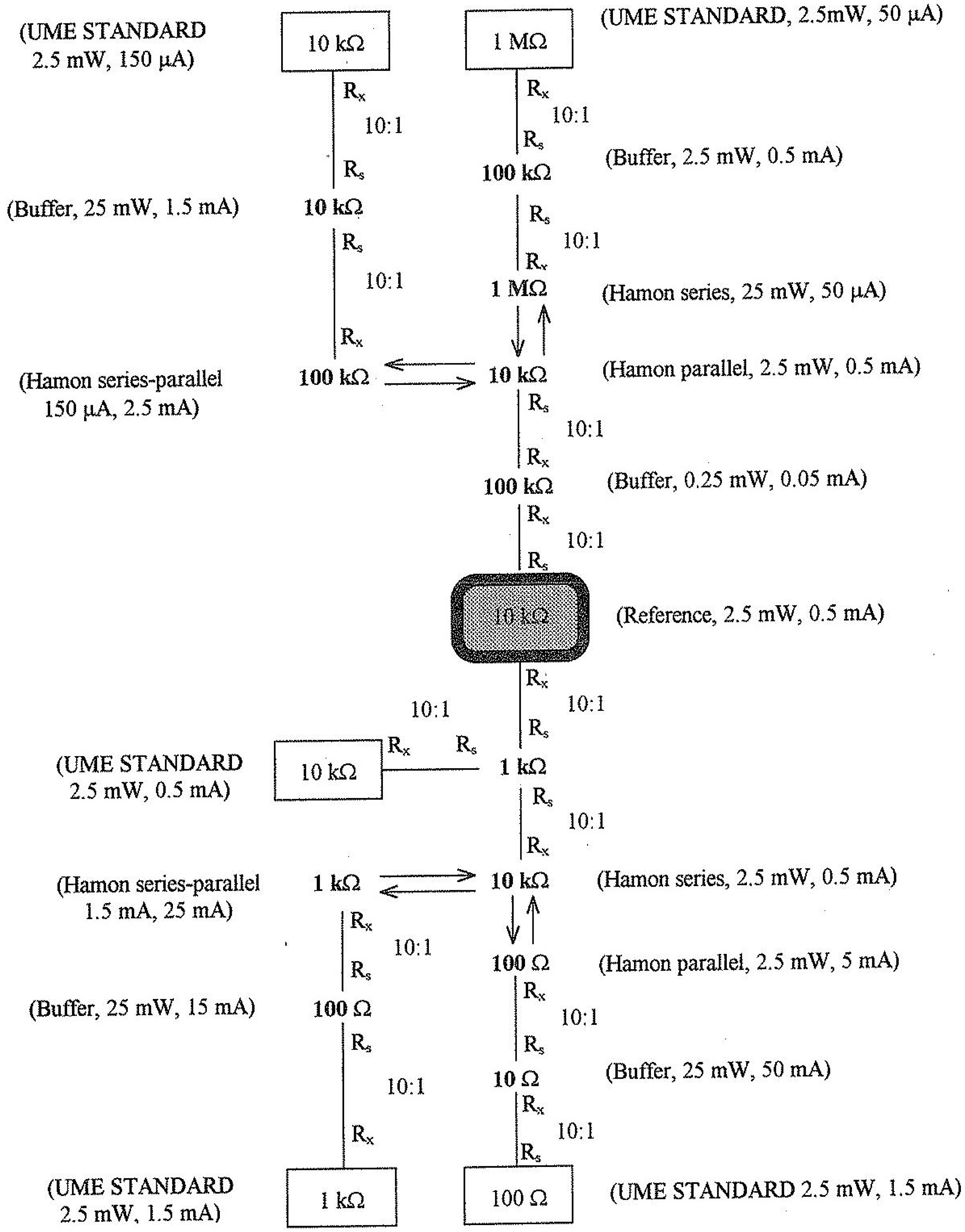

Figure 14. Resistance calibration chain from 100 Ω to 1 MΩ